DEVELOPMENT OF MATHEMATICS IN INDIA

K. Ramasubramanian
IIT Bombay

Course: HS 636

Vedas and Sulbasūtras - Part 2

Outline

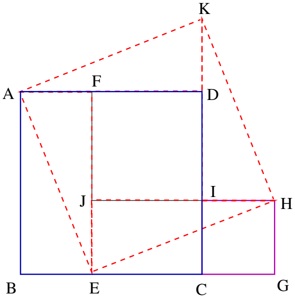
- Sum of unequal squares & implication of the procedure?
- A note on the terminology employed (*karaṇī*)
- Applications of *Śulba* theorem
  - Constructing a square that is difference of two squares
  - Transforming a rectangle into a square
  - To construct a square that is *n* times a given square
- To transform a square into a circle (approx. measure preserving)
- Approximation for $\sqrt{2}$
- *Citi* – Fire altar (types, shapes, etc)
- Fabrication of bricks, Constructional details
- General observations
- References

Constructing a square that is sum of unequal squares

An application of the *Śulba*-theorem

नानाचतुरश्रे समस्यन् कनीयसः करण्या वर्षीयसो वृध्रमुल्लिखेत्। वृध्रस्य अक्ष्णयारज्जुः समस्यतोः पार्श्वमानी भवति। (BSS I.50)

Desirous of combining different squares, may you mark the rectangular portion of the larger [square] with a side (*karaṇyā*) of the smaller one (*kanīyasaḥ*). The diagonal of this rectangle (*vṛddhra*) is the side of the sum of the two [squares].



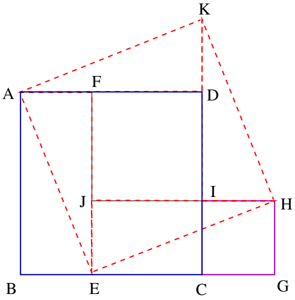
- The term *vṛdhra* in the above *sūtra* refers to the rectangle ABEF.
- Asking us to mark this rectangle, the text states that the cord AE *akṣṇayārajjuḥ* gives the side of the sum of the squares.
- In other words,

$$\begin{aligned} AE^2 &= ABCD + CGHI \\ &= AB^2 + CG^2 \\ &= AB^2 + BE^2. \end{aligned}$$

Implication of the above construction ?

The contruction described clearly proves that *Śulbakāras* knew the proof

- Scholars trained in the Euclidean tradition, puzzled by the mere statement of theorem, without the so called 'proofs' always wondered whether the Śulbakāras knew the proof of *Śulba*-theorem, or was it purely based on empirical guess work?
- Though *Śulvakāras* do not give "explicit" proofs, it is quite implicit in the procedures described by them. In fact, the prescription for combining squares given in the previous *sūtra* clearly forms an example of that.



- In the figure, ABCD and CGHI are the two squares to be combined. E is a point on BC such that $CG = BE$.
- ABEF is the rectangle that is formed. Now the sum of the two squares may be expressed as

$$\begin{aligned} ABCD + CGHI &= ABE + AEF + EHJ + HEG + FDIJ \\ &= ADK + AEF + EHJ + HKI + FDIJ \\ &= AEHK, \end{aligned}$$

which unambigously proves the theorem.

A note on the terminology employed

- Before introducing *Śulva*-theorem, Kātyāyana has exclusively devoted one ***sūtra*** to clarify the different terminologies that would be employed to refer to cords in different contexts.

  करणी, तत्करणी, तिर्यङ्मानी, पार्श्वमानी, अक्ष्णया चेति रज्जवः।
  karaṇī, tatkaraṇī . . . all refer to cords. *(KSS 2.3)*

- The commentary by Mahīdhara (c. 1589 CE)—explaining the origin of the five names given in the above *sūtra*—is quite edifying.

  करणी क्रियते क्षेत्रपरिच्छेदः अनयेति करणी।

  That which limits or produces the length or area is ***karaṇī*** (producer).

  तत्करणी तत्क्षेत्रं द्वैगुण्यादि क्रियते अनया सा तत्करणी, द्विकरणी, त्रिकरणी, चतुःकरण्यादिः।

  That which produces an area that is twice etc. is called ***tatkaraṇī*** (that-producer); For example, *dvikaraṇī, trikaraṇī, catuḥkaraṇī*, and so on.

A note on the terminology employed (contd.)

तिर्यङ्मानी तिर्यक् श्रोण्यंशस्वरूपं मीयतेऽनयेति सा तिर्यङ्मानी, प्राचीसूत्रान्तयोः तिर्यग्वर्तमानं रज्जुद्वयम्।

That by which ... is measured is called *tiryaṅmānī* (transverse-measurer) ...

पार्श्वमानी पार्श्वं मीयतेऽनया सा पार्श्वमानी, पार्श्वयोर्वर्तमानं पूर्वापरायतं रज्जुद्वयम्।

That by which the sides are measured is called *pārśvamānī* (side-measurer); It refers to the cords on either sides that is stretched along the east-west direction.

अक्ष्णया अक्षिवत् क्षेत्रं नयतीति अक्ष्णया, कोणसूत्रभूता मध्यरज्जुः, तस्यां दत्तायां चतुरस्रं अक्षिद्वयसदृशं भवति, ततोऽक्ष्णयेति कोणसूत्ररज्जुः।

That which makes the area look like eyes [i.e., splits into two] is called *akṣṇayā* (diagonal); The mid-cord that connects the corners. Once it is fixed, the square looks like an eye, and hence the term *akṣṇayā* is used to refer to the diagonal.

Different connotations of the word *karaṇī*

1. करणी = side of a square

   कनीयसः करण्या वर्षीयसो वृध्रमुल्लिखेत्।
   By the side of the smaller [square] . . . *(BSS 2.1)*

2. करणी = square root

   पदं तिर्यङ्मानी त्रिपदा पार्श्वमानी तस्य अक्ष्णयारज्जुः दशकरणी।
   [In a rectangle] with upright one pada and base three padas, the diagonal-rope is $\sqrt{10}$. *(KSS 2.4)*

3. करणी = a certain unit of measure

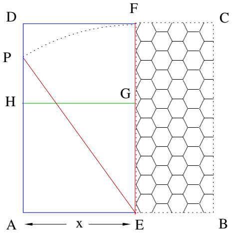
   करणीं तृतीयेन वर्धयेत्, तच्चतुर्थेन, आत्मचतुस्त्रिंशेनोनेन, सविशेषः इति विशेषः। *(KSS 2.9)*

Note: Though *karaṇī* seems to have 'different' connotations, on taking a closer look, it becomes evident that some of these meanings converge to the same thing—*that which makes a square of area a*. Obviously *'that'* $= \sqrt{a}$. Examples *dvikaraṇī*, *trikaraṇī*, *daśakaraṇī*, and so on.

Constructing a square that is difference of two squares

चतुरश्राच्चतुरश्रं निर्जिहीर्षन् यावन्निर्जिहीर्षेत् तस्य करण्या वर्षीयसो
वृध्रमुल्लिखेत्। वृध्रस्य पार्श्वमानीम् अक्ष्णया इतरत् पार्श्वं उपसंहरेत्। सा यत्र
निपतेत् तदपच्छिन्द्यात्। छिन्नया निरस्तम्। *[BSS 2.2]*

Desirous of subtracting a square from another square, may you mark the rectangular portion of the larger [square] with a side ($karaṇyā$) of the smaller one that you want to remove. With the [cord corresponding to the larger] side of the rectangle turned into a diagonal ($akṣṇayā$) touch the other side. Wherever that intersects, chop off that portion. Whatever remains after chopping, gives the measure of the difference.



- *Problem*: Find the side of square which is the difference of the squares ABCD and AEGH.
- *Solution*: Obtain the $vṛdhra$ (rectangle) AEFD, and with radius EF mark a point P on AD. AP gives the desired measure.
- It is evident from the figure

$$\begin{aligned} AP^2 &= EP^2 - AE^2 \\ &= AD^2 - AE^2. \qquad (EP = AD) \\ &= ABCD - AEGH \end{aligned}$$

Transforming a rectangle into a square

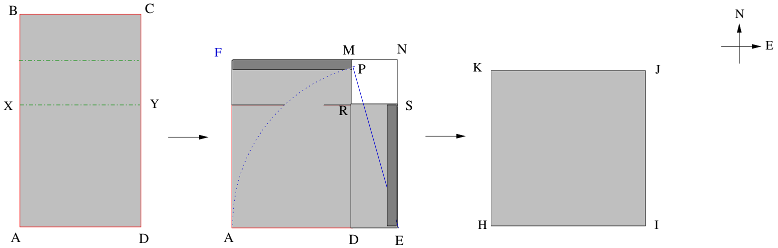
Sequel to finding the sum and difference of squares

दीर्घचतुरश्रं समचतुरश्रं चिकीर्षन् तिर्यङ्मानीं करणीं कृत्वा शेषं द्विधा विभज्य, पार्श्वयोरुपदध्यात्। खण्डम् आवापेन तत्संपूरयेत्, तस्य निर्हार उक्तः।
[BSS 2.5]

terms in *sūtra*	correspondence with figure
दीर्घचतुरश्रं	rectangle ABCD
तिर्यङ्मानी	east-west cord (AD)
शेषम्	the portion XYCB
खण्डम्	square RSNM
आवापेन	by placing

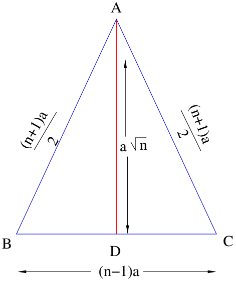
It is evident from the figure

$$\begin{aligned} DP^2 &= EP^2 - DE^2 \\ &= AE^2 - RS^2. \\ &= AENF - RSNM \\ &= HIJK \end{aligned}$$



To construct a square that is *n* times a given square

- Kātyāyana gives an ingenious method to construct a square whose area is *n* times the area of a given square.



यावत्प्रमाणानि समचतुरश्राणि एकीकर्तुं चिकीर्षेत्
एकोनानि तानि भवन्ति तिर्यक् द्विगुणान्येकत
एकाधिकानि त्र्यस्रिर्भवति। तस्येषुः तत्करोति। [KSS 6.7]

As much . . . one less than that forms the base . . . the arrow of that [triangle] makes that (gives the required number $\sqrt{n}$).

Note: Here एकतः (= एकपार्श्वगतानि) द्विगुणानि।

F

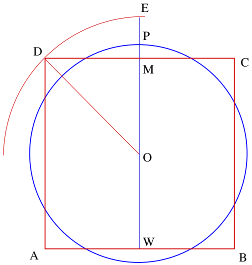
- In the figure $BD = \frac{1}{2}BC = (\frac{n-1}{2})a$. Considering $\triangle ABD$,

$$
\begin{aligned}
AD^2 &= AB^2 - BD^2 = \left(\frac{n+1}{2}\right)^2 a^2 - \left(\frac{n-1}{2}\right)^2 a^2 \\
&= \frac{a^2}{4}[(n+1)^2 - (n-1)^2] = \frac{a^2}{4} \times 4n = na^2
\end{aligned}
$$

Corollary of Kātyāyana's prescription

- Some scholars opine that the statement of "Pythagorean" theorem, could simply be based on empirical knowledge, as the *Śulbakāras* have not given any proof.
- Can the geometrical construction given by Kātyāyana simply arise out of mere empirical knowledge?
- The succinct description only means that the Śulbakāras could do fairly 'sophisticated' mathematics—of turning an algebraic equation into a beautiful geometrical construction.
- In other words, an "unstated" algebraic principle, has been cleverly applied to solve a very practical problem – of finding the value of $\sqrt{n}$ – by resorting to constructive geometry.
- This is indeed sophisticated considering the fact that genesis of algebra was yet to take place in a formal way centuries later.
- Bottomline: Validity of 'Pythagorean theorem'! Did they know?

To transform a square into a circle



चतुरश्रं मण्डलं चिकीर्षन् अक्ष्णयार्धं मध्यात् प्राचीम् अभ्यपातयेत् यद्यदतिशिष्यते तस्य सह तृतीयेन मण्डलं परिलिखेत्। [BSS 2.9]

अक्ष्णयार्धं = semi-diagonal (OD)
मध्यात् प्राचीम् = from centre to the east
यद्यदतिशिष्यते = whatever [portion] remains
तस्य सह तृतीयेन = with one-third of that

$$
\begin{aligned}
AB &= 2a \text{ (given)} \\
OP &= r \text{ (to find)} \\
OD &= \sqrt{2}\, a \\
ME &= OE - OM \\
&= \sqrt{2}\, a - a \\
&= a(\sqrt{2} - 1)
\end{aligned}
$$

As per the prescription given,

$$
\begin{aligned}
\text{Radius } OP = r &= a + \frac{1}{3} ME \\
&= a\left[1 + \frac{1}{3}(\sqrt{2} - 1)\right] \\
&= \frac{a}{3}(2 + \sqrt{2}).
\end{aligned}
$$

How to find $\sqrt{2}$?

How did *Śulvakāras* specify the value of $\sqrt{2}$?

- The following *sūtra* gives an approximation to $\sqrt{2}$:

  प्रमाणं तृतीयेन वर्धयेत्, तच्चतुर्थेन, आत्मचतुस्त्रिंशोनेन, सविशेषः। *[BSS 2.12]*

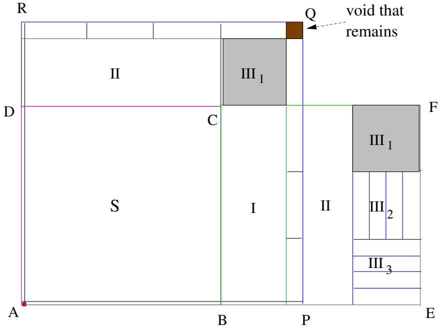
$$\sqrt{2} \approx 1+\frac{1}{3}+\frac{1}{3\times 4}\left(1-\frac{1}{34}\right) \quad (1)$$
$$= \frac{577}{408}$$
$$= 1.414215686$$

- What is noteworthy here is the use of the word सविशेषः in the *sūtra*, which literally means 'that which has some speciality' (speciality $\equiv$ being approximate)
- How did the *Śulvakāras* arrive at (1)?
- Several explanations have been offered by scholars. Here we will discuss the geometrical construction approach.

Approximation for $\sqrt{2}$

Rationale for the expression $\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3.4} - \frac{1}{3.4.34}$ by Geometrical Construction

- Consider two squares *ABCD* and *BEFC* (sides of unit length).
- The second square *BEFC* is divided into three strips.
- The third strip is further divided into many parts, and these parts are rearranged (as shown) with a void at *Q*.
- Now, each side of the new square $APQR = 1 + \frac{1}{3} + \frac{1}{3.4}$.



Approximation for $\sqrt{2}$

Rationale for the expression (contd.)

- The area of the void at Q is $\left(\frac{1}{3.4}\right)^2$.
- Suppose we were to strip off a segment of breadth b from either side of this square, such that the area of the stripped off portion is exactly equal to that of the void at Q, then we have,

$$2b\left(1+\frac{1}{3}+\frac{1}{3.4}\right)-b^2=\left(\frac{1}{3.4}\right)^2.$$

- Neglecting b^2 (as it is too small), we get

$$b=\left(\frac{1}{3.4}\right)^2\times\frac{3.4}{34}=\frac{1}{3.4.34}.$$

- Hence the side of the resulting square

$$1+\frac{1}{3}+\frac{1}{3.4}-\frac{1}{3.4.34}\approx\sqrt{2}$$

Approximate value of π

An estimate of the value of π used by *Śulvakāras*

- If $2a$ is the side of the square, then we saw that the prescription given in the text amounts to taking the radius of the circle to be

$$r = a\left[1 + \frac{1}{3}(\sqrt{2} - 1)\right] \tag{2}$$

- If we were to impose the constraint that the transformation has to be measure preserving, then it translates to the condition $\pi r^2 = 4a^2$.
- From the relation given above we have,

$$\pi\left[\frac{1}{3}(2 + \sqrt{2})\right]^2 \approx 4. \tag{3}$$

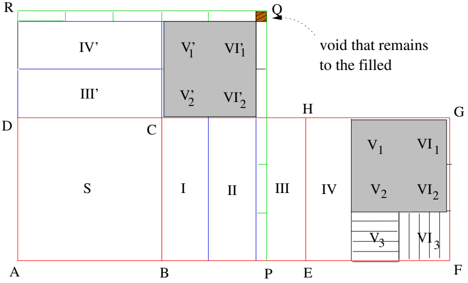
- Using the value of $\sqrt{2}$ given in the text we get

$$\pi \approx 3.0883, \tag{4}$$

which is correct only to one decimal place.

Value of $\sqrt{3}$ (*trikaraṇī*)

Geometrical construction described by Datta



- Each side of the new larger square $APQR = 1 + \frac{2}{3} + \frac{1}{3.5}$
- To obtain a better approximation, let the side of the new square be diminished by an amount y, such that

$$2y\left(1+\frac{2}{3}+\frac{1}{3.5}\right)-y^2=\left(\frac{1}{3.5}\right)^2$$

Neglecting y^2 as too small, we get $y = \frac{1}{3.5.52}$, nearly.

- Thus we get $\sqrt{3} = 1 + \frac{2}{3} + \frac{1}{3.5} - \frac{1}{3.5.52}$

Problem of squaring a circle

मण्डलं चतुरश्रं चिकीर्षन् विष्कम्भम् अष्टौ भागान् कृत्वा[1] भागं एकोनत्रिंशद्धा विभज्य अष्टाविंशतिभागान् उद्धरेद्, भागस्य च षष्ठं अष्टमभागोनम्[2] ।[3] *[BSS 2.10]*

With the desire of turning a circle into a square [with the same area] dividing the diameter into 8 parts . . .

$$2a = \frac{7d}{8} + \left[\frac{d}{8} - \left\{\frac{28d}{8.29} + \left(\frac{d}{8.29.6} - \frac{d}{8.29.6.8}\right)\right\}\right] \tag{5}$$

This may be rewritten as

$$2a = d\left[1 - \frac{1}{8} + \frac{1}{8.29} - \frac{1}{8.29.6} + \frac{1}{8.29.6.8}\right] \tag{6}$$

---

[1] भागमुद्धरेत्,
[2] षष्ठभागो यः, सः तदष्टमभागोनः कार्यः। तम् उद्धरेत्।
[3] (पूर्वस्मात्) भागात् उद्धरेत् – इत्यनुषज्यते।

Citi: Fire altar

- चितिः – Platform constructed of burnt bircks and mud mortar.
- चीयते अस्याम् इति चितिः: [the locus] unto which things are brought into [and arranged].
- चि (संवृत्यादानयोः)= assembling or fetching together
- Fire altars are of two types. The ones used for
  - नित्यकर्म—daily ritual.
  - काम्यकर्म—intended for specific wish fulfilment.
- The fire altars are of different shapes. They include प्रौगचिति (isosceles triangle), उभयतः प्रौगचिति (rhombus), रथचक्रचिति (chariot wheel), द्रोणचिति (a particular type of vessel/water jar), कूर्मचिति (tortoise), श्येनचिति (bird, falcon type), etc.
- Number of bricks used is 1000 (साहस्रं चिन्वीत ...), 2000, and 3000.
- Altar has multiples of five layers, with 200 bricks in each layer.

Types of Fire altars (representative list)

- Different types of wish-fulfilling fire-altars are described in Vedas.

  छन्दश्चितं चिन्वीत पशुकामः पशवो वै छन्दाꣳसि पशुमानेव भवति, श्येनचितं चिन्वीत स्वर्गकामः श्येनो वै वयसां प्रतिष्ठा श्येन एव भूत्वा स्वर्गं पतति . . . प्रौगचितं चिन्वीत भ्रातृव्यवान् प्रैव भ्रातृव्यान् नुदते, . . . रथचक्रचितं चिन्वीत ग्रामकामः . . .

- The table below presents a list some of them, along with the shapes and the purpose as stated in the text.

Name of the *citi*	Its shape	Who has to perfom
छन्दश्चिति	Form of a bird	Desirous of cattle
श्येनचिति, कङ्कचिति	Form of bird	Desirous of heaven
प्रौगचिति	Isoceles triangle	Annihilation of rivals
रथचक्रचिति	Chariot wheel	Desirous of region
द्रोणचिति	Form of a trough	Abundance in food

Table : Different *citis*, their shapes and purpose.

On the height and the shape of *citis*

Measurements were case-based (based on the performer) and not ‘standardized’

- *Taittirīya-saṃhitā*, prescribing the height of the *citi* observes:[4]

  जानुदघ्नं चिन्वीत प्रथमं चिन्वानः, गायत्रियैवेमं लोकमभ्यारोहति,
  नाभिदघ्नं चिन्वीत द्वितीयं चिन्वानः त्रिष्टुभैवान्तरिक्षमभ्यारोहति,
  ग्रीवादघ्नं चिन्वीत तृतीयं चिन्वानः, जगत्यैवामुं लोकमभ्यारोहति।

  Knee-deep should he pile when he piles for the first time, and indeed he mounts this world with gāyatrī, naval-deep should he pile when he piles the second time, . . . neck-deep should he pile when he piles the third time . . .

- Elsewhere (5.5.3) observing on the shape of the fire-altar it is said that it should be akin to the shadow cast by the bird.

  वयसां वा एष प्रतिमया चीयते यदग्निः। यन्न्यञ्चञ्चिनुयात्[5]

[4] *Taittirīya-saṃhitā* 5.6.8.

[5] वयसां वा एष प्रतिमया चीयते उत्पततां छाययेत्यर्थः (BSS 8.5)

Śyenaciti—Falcon-shaped fire-altars

- The origin of *Śyenaciti* can be traced back to *vedas*.
- For instance in *ṣaḍviṃśa brāhmaṇa* belonging to *sāmaveda*, अथैष श्येन …यथा श्येन आदधीत एवमेव एनमेतेन आदत्ते।[6]
- Another version of the same statement perhaps on another *Brāhmaṇa* which is more popular goes as यथा वै श्येनो निपत्य आदत्ते एवमेवायं द्विषन्तं भ्रातृव्यं निपत्य आदत्ते।
- These sentences are cited in the *Mīmāṃsā* text in connection with the discussion on deciding the meaning of the word *śyena* that appears in the *vidhi* (injunction)

  श्येनेनाभिचरन् यजेत।

[6] *Ṣaḍviṃśa-brāhmaṇa* 4.2.3.

Measurement units used in construction

अथाङ्गुलप्रमाणम्। चतुर्दशाणवः। चतुस्त्रिंशत्तिलाः पृथुसंश्लिष्टा इत्यपरम्। दशाङ्गुलं क्षुद्रपदम्। द्वादश प्रादेशः। पदं पञ्चदश। द्विपदः प्रक्रमः। द्वौ प्रादेशावरत्निः। पञ्चारत्निः पुरुषः। चतुररत्निर्व्यायामः।[7]

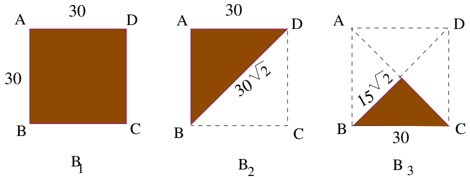
$$\begin{aligned}
a\dot{n}gula &= 14\ a\d{n}u \text{ or } 34\ tila \\
k\d{s}udrapada &= 10\,a\dot{n}gula \\
pr\bar{a}de\acute{s}a &= 12\,a\dot{n}gula \\
pada &= 15\,a\dot{n}gula \\
prakrama &= 30\,a\dot{n}gula \\
aratni &= 2\,pr\bar{a}de\acute{s}a = 24\,a\dot{n}gula \\
vy\bar{a}y\bar{a}ma &= 4\,aratni \\
puru\d{s}a &= 5\,aratni
\end{aligned}$$

[7] *Baudhāyana-śulbasūtra*,1.3

Construction of *Śyenaciti* (Type 1)

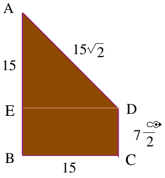
Types of bricks: 1, 2 and 3

- Bricks of geometrical shapes other than rectilinear are needed.
- The five types of bricks used:
    1. B_1—one-fourth brick (*caturthī*)—30×30 *aṅgulas*; i.e., a square whose side is $\frac{1}{4}$ *pu*.
    2. B_2—half brick (*ardhā*)—obtained by cutting the one-fourth square brick diagonally; each of 2 sides equals *aṅgulas* and the hypotenuse 30 $30\sqrt{2}$ *aṅgulas*
    3. B_3—quarter brick (*pādyā*)—obtained by cutting B_1 diagonally; each of 2 sides equals $15\sqrt{2}$ *aṅgulas* and hypotenuse 30 *aṅg*.

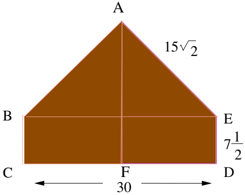


Types of bricks: 4 and 5

- B_4—four-sided quarter brick (*caturaśra-pādyā*)—of sides equal to $22\frac{1}{2}$, 15, $7\frac{1}{2}$ and $15\sqrt{2}$ *aṅg*. The area is 15×15 *aṅg*, the same as that of B_3.
- B_5—(*haṃsamukhī*)- half brick obtained by joining two B_4s, along their common longest side.

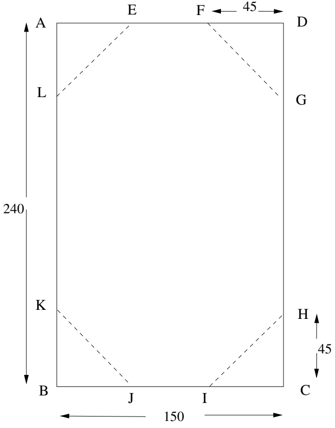


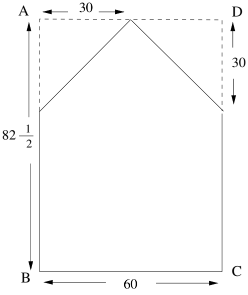
B 4



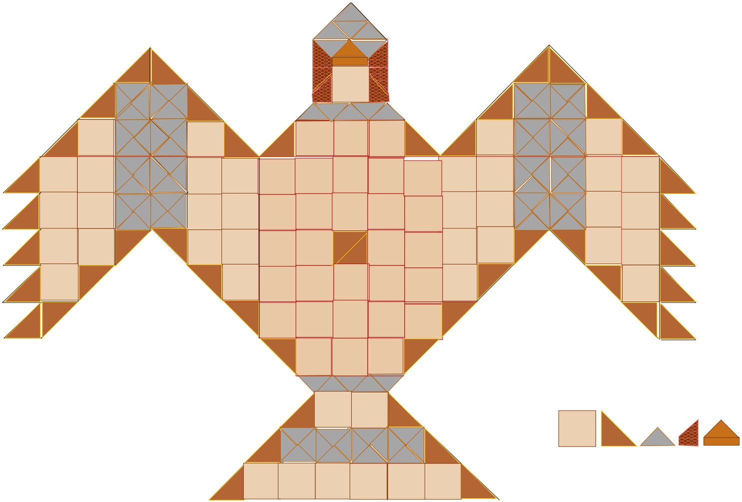
B 5

Outline of body and head of *Śyena*





Śyenaciti: Falcon-shape



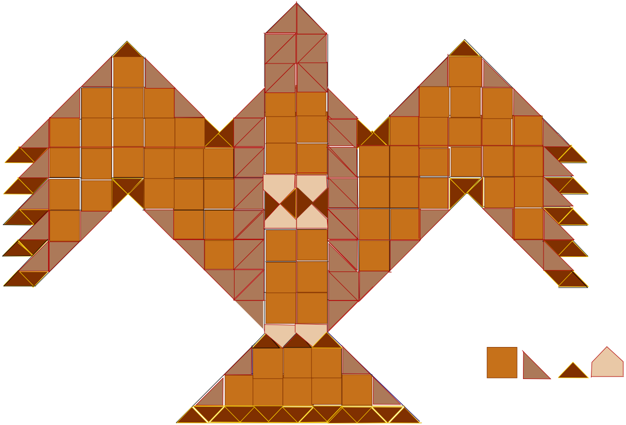
Number of bricks used

एवं षड्चत्वारिंशदात्मनि। शिरसि चतुर्दश। द्वात्रिंशत्पुच्छे। पक्षयोरष्टशतम्।
अस्मिन् प्रस्तारे नवषष्टिश्चतुर्थ्यः। अर्धा द्वासप्ततिः पाद्या द्विपञ्चाशत्।
षट् चतुरश्रपाद्या। एका हंसमुखी।

Parts of the *citi*	B_1	B_2	B_3	B_4	B_5	Total
Head	1		6	6	1	14
Body	30	6	10			46
Wings	30	62	16			108
Tail	8	4	20			32
Total	69	72	52	6	1	200

Śyenaciti: second layer

Number of bricks used in the second layer



Parts	B_1	B_2	B_3	B_4	B_5	Total
Head		10				10
Body	12	28	4		4	48
Wings	48	28	34			110
Tail	8	4	18		2	32
Total	68	70	56		6	200

Fabrication of bricks

Ingredients to be added to the mixture of clay employed in manufacting the bricks

- पर्णकषायनिष्पक्वा एता आपो भवन्ति। स्थेम्ने न्वेव।...
- Extracts of gum from certain trees (*palāśa*)
- अथ अजलोमैः संसृजति। स्थेम्ने न्वेव।...
- Hair of the goat, of a bullock, horse, etc.
- शर्कराश्माहो रसः तेन संसृजति। स्थेम्ने न्वेव।[8]
- Fine powder of burnt bricks ..
- उख्यभस्मना संसृज्य इष्टकाः कारयेदिति। संवत्सरभृतः एव एतदुपपद्यते। न रात्रिभृतः।[9]

The above process of strengthening is in practice till date.[10]

---

[8] *Śatapatha brāhmaṇa*, 6.5.1.1–6.

[9] *Baudhāyana Śulbasūtra*, 2.78–79

[10] The addition of fly ash as well as pozzuolana is well known in the manufacture of cement.

Fabrication of bricks

Handling the contraction in size of the brick (Sun's heat + Burning in the kiln)

- There will be reduction in the size of the moulded bricks:
  - इष्टका शोषपाकाभ्यां त्रिंशन्मानात्तु हीयते।[11]
- Different *Śulbasūtra* texts suggest different measures to handle this problem of contraction
  - सदा च त्रिंशकं भागं इष्टका ह्रसते कृता।
    तावत् समधिकं कार्यं करणं सममिच्छता॥[12]
  - Appropriately increase the size of the mould.
  - यच्छोषपाकाभ्यां प्रतिह्रसेत् पुरीषेण तत् संपूरयेत्, पुरीषस्य अनियतपरिमाणत्वात्।[13]
  - Compensate the loss with the mortar.

[11] *Mānava Śulbasūtra*, 10.3.4.17
[12] *Mānava Śulbasūtra*, 10.2.5.2
[13] *Baudhāyana Śulbasūtra*, 2.60

Constructional Details

Specifications regarding the arrangement of bricks in different layers

- भेदान् वर्जयेत्।
  - Here the word “*bheda*” does not simply mean difference/distinction (in fact, this has to be maintained).
  - What is meant is a clear segregation between two rows across all the layers. This is to be avoided.
  - Joints should be disjoint! (not continuous)
- अधरोत्तरयोः पार्श्वसन्धानं भेदा इति उपदिशन्ति।[14]
  - The etymology could be: भेदहेतुभूतत्वात् भेदः।
- अमृन्मयीभिः अनिष्टकाभिः न सङ्ध्यां पूरयेत्।
  - (Arbitrary) foreign material should not be employed to fill the gaps.

The above-mentioned are very important principles from the view point of civil engineering.

[14] BSS. 2.22–23. (RPK’s Book)

General observations

- The purpose for which the geometry got developed in the Indian context is construction and transformation of planar figures.
- We saw that Bodhāyana (prior to 800 BCE) not merely listed the so-called 'Pythagorean' triplets, but also gave the theorem in the form of an explicit statement.
- Extensive applications of the theorem in the context of scaling and transformation of geometrical figures was also discussed.
- Though *Śulbakāras* did not explicitly give proofs—which anyway was NOT a part of their "oral" tradition (of the antiquity)—it is evident from several applications discussed, that the proof is implicit.
- From the view point of history it may also be worth recalling:

  Antiquity? Though the Babylonians of 2nd millenium BCE had listed triplets in cuneiform tablets, there is no general statement in the form of a theorem.

  Pythagorean? Since there is hardly any evidence to show Pythagoras himself was the discoverer of the theorem, some of the careful historians call it Pythagorean theorem.

General observations

- It may be reiterated that *Śulbasūtra* texts were primarily meant for assisting the Vedic priest in the construction of altars designed for the performance of a variety of sacrifices.
- However, these texts shed a lot of light from the view point of development of mathematics in the antiquity, particularly the use of arithmetic and albegra, besides geometry.

  Use of fractions The expressions used by the *Śulvakāras* for expressing surds—in terms of sums of fractions, leading to a remarkable accuracy[15]—is quite interesting.

  Use of algebra The rules and operations described by them in the context of scaling geometrical figures unambiguously demonstrate the use of algebraic equation.
- The different *citis* not only speak of the aesthetic sense, but also of the creativity and ingenuity of *Śulvakāras* to work with several constraints imposed—both in terms of area and volume.
- The archaeological excavations at Kausambi (UP) seems to have revealed a *Śyenaciti* constructed around 200 BCE.

[15] Five decimal places in the case of $\sqrt{2}$.

References

- *Kātyāyana-śulbasūtram*, with the comm. of Karka and Mahīdhara, Kashi Sanskrit Series, No. 120. (1900?) [digital version from Univ. of Toronto].
- B. B. Datta and A. N. Singh, 'Hindu Geometry', *Indian Journal of History of Science*, Vol. 15, No. 2, 1979, pp. 121–188.
- S. N. Sen and A. K. Bag, *The Śulbasūtras*, INSA, New Delhi 1983.
- Bibhutibhushan Datta, *Ancient Hindu Geometry, The Science of Śulba*, Cosmo Publications, New Delhi 1993.
- T. A. Sarasvati Amma, *Geometry in Ancient and Medieval India*, Motilal Banarsidass, New Delhi 1979; Rep. 2007.

Thanks!

THANK YOU