CC-13

Advances in Indian Astronomy - A Historical Perspective

K.Ramasubramanian
Department of Humanities of Social Sciences
IIT Bombay
Powai, Mumbai 400 076, India
e-mail: ram@hss.iitb.ac.in

July 15, 2004

1 Introduction

It is quite interesting to note that there has been a continuous tradition of astronomy in India right from the period of *Vedāṅga Jyotiṣa* (14th cent. BC) to the *Siddhānta darpaṇa* of Samanata Chandra Sekhara (19th cent. AD). The Indian astronomers have been continously refining their parameters and procedures in order to achieve better accuracy in their predictions of the positions of the sun, moon and planets. Their positions are indicated at regular intervals in Indian calendar called *Pañcāṅga*. This *Pañcāṅga* was, and is still today, used by almost every section of the society for observance of social, religious, socio-religious and cultural functions. The relation between Veda, the science of astronomy and the purpose they serve in the society are beautifully brought out by the following verse[1]:

> ***vedāstāvad yajñakarmapravṛttāḥ yajñāḥ proktāḥ te tu kālāśrayeṇa |***
> ***śāstrādasmāt kālabodho yataḥ syāt vedāṅgatvaṃ jyotiṣasyoktamasmāt ||***

The *Vedas* are meant for prescribing sacrifices (duties), and the duties are to be perfomed at appropriate times. The science of astronomy gives the knowledge of time; hence it has been considered as one of the *Vedāṅgās*.

This verse succintly points out the need for a precise calendar. The astronomers took upon this as a challenge and a noble task entrusted upon them. This has been the primary motivation for the developments and innovative formulations in Indian astronomy.

2 Early phase of Indian Astronomy

The earliest astronomical text *per se*, extant is the *Vedāṅga Jyotiṣa* (c.1400 BC) composed by Lagadha. Here a cycle of five years is conceived to be constituting a *yuga*, in which there would be five complete revolutions of the sun, 67 revolutions of the moon and 62 synodic months *cāndra-māsa*. This *yuga* consists of 1830 civil days; 1860 lunar days or *tithis*. This amounts to taking the duration of a solar year to be 366 days, equally divided into two *ayanās* consisting of 183 days and 6 *ṛtus* each of 61 days duration. The number of solar months in a period five years consisting of 1830 days is 60, whereas the number of lunar months in the same period consisting of 1860 lunar days *tithis* is 62 ($\frac{1860}{30} = 62$). Since, the Indians were following a luni-solar calendar, two lunar months had to be discarded from the calendar in every *yuga*, and they were called *Adhikamāsas* (intercalary months). We find the names of the twelve regular months and two intercalary months even in the vedic literature [2]. The text *Vedāṅga Jyotiṣa* also clearly mentions the name of 27 *nakṣatras* and smaller units of time like *muhūrtas* [3].

While this calendar system has been in vogue for a long time, possibly a few centuries before the dawn of Christian era, a new class of astronomical literature started emerging called the *siddhāntas*.

3 Siddhantic Astronomy

The earliest astronomical treatise of this class available to us is *Aryabhaṭīya* of Aryabhata (c.499 AD). The mathematical astronomy presented in this text is a culmination of the knowledge base developed over centuries before him. As Aryabhata himself mentions towards the end of his seminal work [4]:

> *sadasajjñānasamudrāt samuddhṛtaṃ devatāprasādena* |
> *sajjñānottamaratnaṃ mayā nimagnaṃ svamatināvā* ||

> With the help of the divine grace, and by my own intellect in the form of a boat, I have lifted up (traced) the gem of right knowledge, from the ocean of the right and the false knowledge.

It is through the work of Varahamihira (c. 505 AD), the junior contemporary of Aryabhata, we understand that the siddhantic tradition must have originated at least with the advent of Christian era, if not earlier. Having choosen five popular ancient *siddhāntas* extant during his time, Varahamihira has compiled them into a treatise called *Pañcasiddhāntikā* [5]. The *siddhāntas* which form part of this classical treatise, in chronological order, are: (i) *Paitāmaha Siddhānta*, (ii) *Vasiṣṭha Siddhānta* (iii) *Pauliśa Siddhānta*, (iv) *Romaka Siddhānta* (v) *Sūrya/Saura Siddhānta*.

These five *siddhāntas*, exemplify the continuous progress that has been taking place since the advent of Christian era till the end of 5th cent. AD. Beginning from *Vasiṣṭha Siddhānta*, one observes a gradual advancement in the matter of (i) specification of the duration of a year, (ii) periods of solar months, (i.e., the actual time taken by the sun to cross over 30 degrees of ecliptic called *rāśi*). (iii) the revolution periods of the planets, (iv) the procedures adopted for the computation of true longitudes of the sun, moon and the planets, (v) the application of equation of centre, (vi) the determination of the geocentric latitude of a planet, and so on.

In *Vedāṅga Jyotiṣa*, there is no mention of the period of individual solar months and the year was taken to be of 366 days. It is for the first time in *Vasiṣṭha Siddhānta* we find the period of solar months being specified in fractions, beginning from the month *Meṣa*. They are given to be $31\frac{1}{4}$, $31\frac{1}{2}$, $31\frac{1}{2}$, $31\frac{1}{2}$, 31, $30\frac{1}{4}$, $29\frac{3}{4}$, $29\frac{1}{4}$, $29\frac{1}{4}$,

$29\frac{1}{2}$, 30, and $30\frac{1}{2}$, which amounts to considering 365.25 days in an year. This is a significant advance over the calendar we find in *Vedāṅga Jyotiṣa*. We also find the discussion of crude form equation of centre for some planets in this text.

Subsequently, in *Pauliśa Siddhānta* we find improved formulation of the equation of centre, paving way for further refined formulations with parameter values found in *Romaka Siddhānta*. Of the five *siddhānta* texts in *Pañcasiddhāntikā*, *Surya Siddhānta* could be considered as the culmination of earlier investigations. Apart from more precise mathematical formulation of the problems, here we find a clear distinction made between different types of motion, namely the *manda* (slow), *śīghra* (fast), *ṛju* (direct) and *vakra* (retrograde) motions observed in the planets. This irregularity in motion was of course ascribed to the extra-normal forces generated by the invisible deities. This is quite understandable, given the cultural background of the siddhantic formlators and the sophistication available in those times.

4 Advances in Indian Astronomy

From the time of Aryabhata (c.499 AD), there has been an unbroken tradition in Indian astronomy for about 14 centuries. By meticulous observations, the astronomers of this period had been continuously updating their data base and were proposing new parameters and procedures having analysed them thoroughly. New mathematical techniques for achieving better accuracy in their predictions were also developed.

The advances made by the Kerala school of mathematician-astronomers beginning from Madhava of Sangamagrama (c.14th cent.) are truly phenomenal [6]. Some of the remarkable advances are to be found in the areas of (i) the formulation of equation of time (ii) the application of equation of centre for inner planets (iii) finding the exact value of the observer's latitude, (iv) the determination of time from shadow measurements (v) the calculation of the duration of *lagna* at the observer's location, (vi) the computation of eclipses (vii) the transformation of the co-ordinates from the geocentric system to the observer-centric system and (viii) fixing the *Mahāpātas*.

In the follwing, we choose a couple of areas namely, (iii) and (iv) mentioned above, to illustrate the theoretical advances made by the Kerala school of astronomers, particularly Nilakantha Somayaji with regard to the computation of quantities of physical interest. These advances are truly remarkable [7].

5 Determination of latitude

From time immemorial, the determination of the terrestrial latitude of an observer has been one of the fundamental problems of astronomy. It plays a key role in the determination of the sunrise and sunset times at the location of the observer, which in turn is important for civil, navigation and other purposes. Astronomers of different traditions have evolved their own methods and techniques, for its accurate determination.

5.1 Traditional Indian method

The Indian astronomers had evolved a very simple, but quite efficient method for the determination of terrestrial latitude of an obserever. This method can also be employed for the determination of other important quantities of physical importance like time of the day, the cardinal directions at one's own place and so on. The tools employed in this method are very minimal. It basically involves measuring the length of the shadow cast by the sun of an instrument called *śaṅku* .

5.2 The *śaṅku*

It essentially consists of a wooden stick of suitable height and thickness sharpened at one of its edges. A variety of *śaṅkus*, slightly differing in their forms are described in Indian astronomical texts. A typical form of *śaṅku* , described in the following verses [8], is shown in Fig.1.

tale dvyaṅgulavistāraḥ samavṛttodvādaśocchrāyaḥ |
sāradārumayaḥ śaṅkuḥ dvitīyo dvādaśāṅgulaḥ ||

sūcyagraḥ sthūlamūlo'nyaḥ tadutsedhatalāgrayoḥ |

(The first kind of gnomon is) two *aṅgulās* in diameter at the bottom, uniformly circular, twelve *aṅgulās* in height, and made of a strong piece of wood. The second kind of gnomon is twelve *aṅgulās* (in height), pointed at the top, and massive at the bottom (conical).

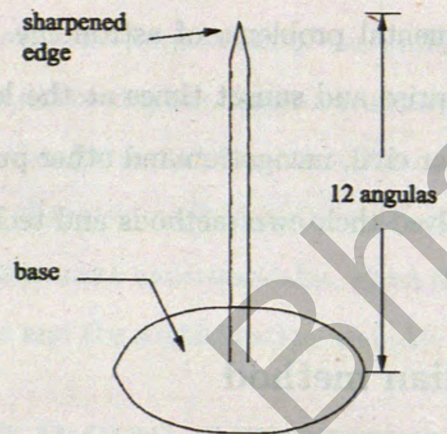

Fig.1: The astronomical device *śaṅku* used for the measurement of latitude.

5.3 Measurement of latitude using *śaṅku*

The principle behind the prescription given in the Indian astronomical texts for the measurement of latitude using a *śaṅku* may be understood with the help of Fig.2. Here OX represents the *śaṅku* (gnomon). On the equinoctial day the diurnal motion of the sun is along the equator throughout the day, ignoring the small change in the declination during the day. Hence, the terrestial latitude (ϕ) is equal to the zenith distance of the sun as it crosses the prime meridian (at noon). On any other day the zenith distance at noon would be $z = \phi - \delta$, where δ is sun's declination.

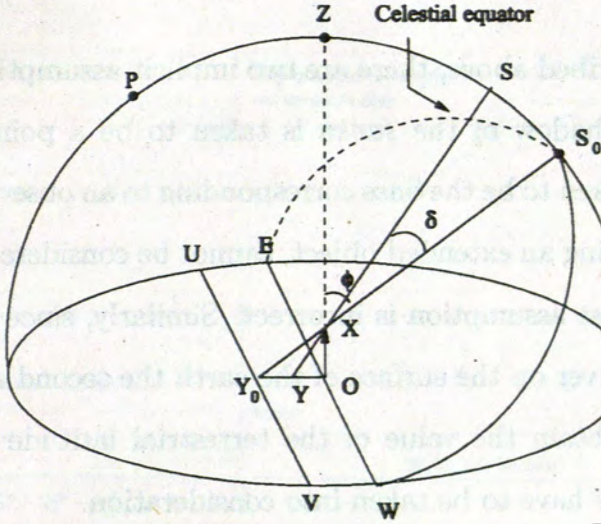

Fig.2: Shadow of *śaṅku* on an arbitrary day (OY) and the equinoctial day (OY_0).

Considering the triangle OXY, formed by *śaṅku* OX, its shadow OY and the hypotenuse XY, it can be easily seen that

$$OY = XY \times \sin(\phi - \delta).$$

On the equinoctial day, $\delta = 0$ and the tip of the shadow Y is Y_0. Hence, the above equation reduces to

$$\sin\phi = \frac{OY_0}{XY_0}.$$

Similarly,

$$\cos\phi = \frac{OX}{XY_0}.$$

Multiplying the above equations by *trijyā* , and using Indian astronomical terms for OX, OY_0 and XY_0, we have

$$ak\d{s}ajy\bar{a} = \frac{trijy\bar{a} \times ch\bar{a}ya}{kar\d{n}a}$$

and,

$$lambaka = \frac{trijy\bar{a} \times \acute{s}a\dot{n}ku}{kar\d{n}a}.$$

The above expressions can be found in almost all the *siddhānta* texts [9].

5.4 Need for refinement

In the procedure described above, there are two implicit assumptions: (i) the source light producing the shadow of the *śaṅku* is taken to be a point source. (ii) the observed values are taken to be the ones corresponding to an observer at the centre of the earth. The sun being an extended object, cannot be considered as a point source of light. Hence the first assumption is incorrect. Similarly, since the measurements are made by the observer on the surface of the earth the second assumption is false as well. Hence, to obtain the value of the terrestrial latitude to high degree of precession their effects have to be taken into consideration.

5.5 Correction due to the finite size of the sun

There is a qualitative difference between the nature of the shadow cast by an extended source of light and a pointed one. We explain this with the help of Fig.3. Here OA represents the *śaṅku* . PSQ represents the sectional view of the sun, where S is its centre, and P and Q are the upper and the lower edges of the solar disc. Rays from S and P grazing the top edge of the *śaṅku* A, meet the plane of the observer's horizon at S' and P' respectively. If the sun were a point source of light, then the

the shadow and the angle $S'\hat{A}O$ would be the terrestial latitude. However, due to the finite size of the sun. P' is the tip of the actual shadow and OP' its length.

$S'\hat{A}O$. $P'\hat{A}S'$ is equal to $P\hat{A}S$, which is the semi-diameter of the sun. Thus, the true latitude of the place is obtained by adding the semi-diameter of the sun to the observed value.

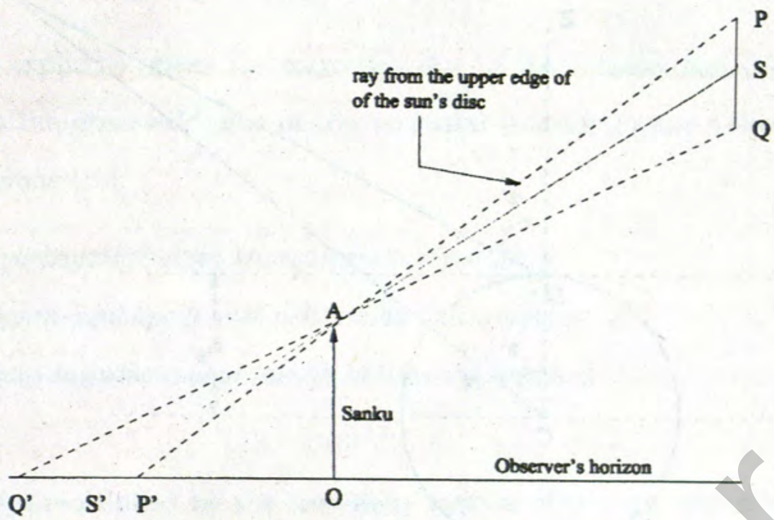

Fig.3: Sectional view of the sun and the shadow of the *śaṅku* generated by it.

5.6 Correction due to the parallax

It is highly desirable to describe the position of an object (terrestrial or celestial) with coordinates which are independent of the position of the observer on the earth. To achieve this, the most natural choice of the reference point would be the centre of the earth. But measurements are, and necessarily have to be made from the surface of the earth. It is here the concept of parallax comes into picture.

We first explain the concept of parallax and its effect on the measurement of the zenith distance of a celestial object using the modern notation and then proceed to discuss its effect as described in Indian astronomical texts. In Fig.4, C represents the center of the Earth, S the celestial object which is chosen for observation, and O the observer. $R_e = OC$ is the radius of the Earth and d is the distance of the sun from the centre of the Earth. Z represents the geocentric zenith of the observer.

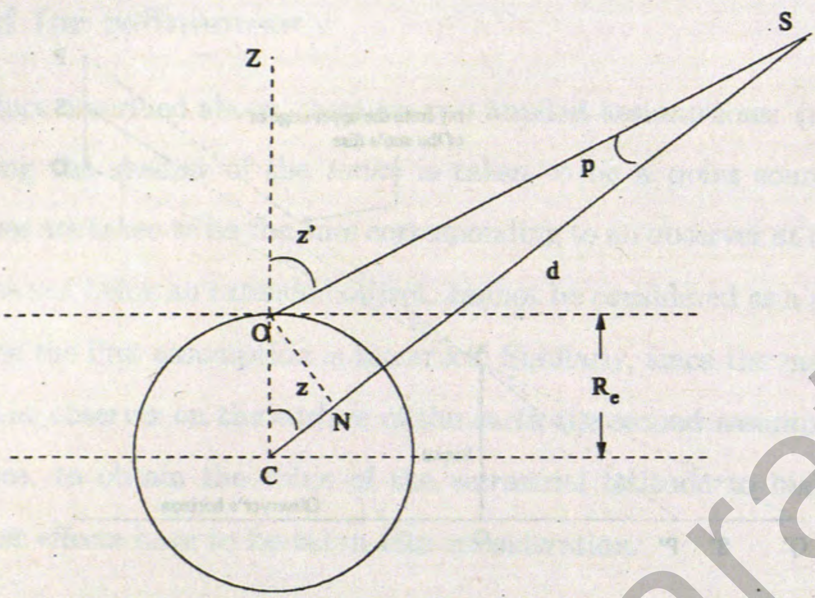

Fig.4: The effect of parallax on the measurement of latitude of the observer

If z' and z represent the apparent (for an observer on the surface of the earth) and the actual (for an observer at the centre of the earth) zenith distances of the sun, then it is easily seen that

$$z = z' - p.$$

where $p = C\hat{S}O$, is the parallax of the celestial object, which is nothing but the angle subtended by the radius of the Earth at the centre of the sun. In other words, it is the angle between the direction of the object as seen by the observer O and the direction of the object as seen from the Earth's centre (which is the standard reference point for measuring the celestial co-ordinates).

As mentioned earlier in Sec(5.3), the measurement of latitude of a place using *śaṅku* is directly related to the measurement of the zenith distance of the sun at the mid-noon. Since the observed value of the zenith distance differs from the actual value, by parallax (p), the observed value of the latitude would also vary from the actual value by the same amount. Therefor, if ϕ' be the observed value of the latitude, then the actual value ϕ is given by $\phi = \phi' - p$.

5.7 Nilakantha's prescription

Nilakantha explicitly states the correction due to the parallax and the finite size of the sun, to the observed value of the terrestial latitude in the following verses of *Tantrasaṅgraha* [10]:

akṣajyārkagatighnāptā khasvareṣvekasāyakaiḥ ||
phalonamakṣacāpaiḥ syāt arkabimbārdhasaṃyutam |
sphuṭaṃ tajjyākṣajīvāpi tasyāḥ koṭiśca lambakaḥ ||

Akṣajyā multiplied by the true daily motion of the sun and divided by 51570 (is the correction factor p). This has to be subtracted from the *akṣacāpa* (latitude of the place). The semidiameter for the sun added to this is the true latitude. The R sine of this is the *akṣajyā* and its complement is the *lambaka* .

Note: In the printed text, the later half of the first line of the above quotation reads as - *khasvarādryekasāyakaiḥ* | The numeral represented by this word is 51770 in *bhūtasaṅkhyā-paddhati.* But the actual figure that occurs in the computation is $15 \times 3438 = 51570$. This number is given by the word *khasvareṣvekasāyakaiḥ* in *bhūtasaṅkhyā.* In a similar context in Chapter V, verse 10, we find the number 51570 occuring again explicitly. There it is given by the word *khasvareṣvekabhūta.* Hence, we suppose that the text must be *khasvareṣvekasāyakaiḥ* and not *khasvarādryekasāyakaiḥ* |

6 Exact expression for time from the shadow

From the shadow of an object cast by the sun, a rough estimate of the time of the day can always be made and this technique has been in vogue from time immemorial. Various other instruments had also been used for time measurements. But an exact theoritical formulation of the problem in terms of spherical astronomy is not found in the Indian astronomical literature till fifteenth century, (and even elsewhere ?). In the third chapter of *Tantrasaṅgraha* , Nilakantha gives the exact procedure for the determination of time from shadow measurements, perhaps for the first time in Indian astronomy. If z is the zenith distance of the Sun at any time, the length of the shadow is $12 \tan z$ at that time. Hence the zenith distance can be obtained from the shadow. Since the latitude of the place ϕ and the declination of the Sun δ are already known, the time can be calculated as described below [11].

vyāsārdhaghnāt tataḥ śaṅkoḥ lambakāptaṃ trijīvayā ||
hatvā dyujyāvibhakte tat carajyā svarṇameva ca |
yāmyodaggolayostasya cāpe vyastaṃ carāsavaḥ ||
saṃskāryā gatagamyāste pūrvāparakapālayoḥ |

The *śaṅku* is multiplied by radius (*trijyā*) and divided by *lambaka* . This if further multiplied by *trijyā* and divided by *dyujyā* . To this quantity (x), the *carajyā* is applied positively or negatively depending upon whether the Sun is in the southern or the northern hemisphere. To the arc of the result, the ascensional difference has to be applied in the reverse order. This gives the time that has elapsed or yet to elapse in the eastern and the western half of the hemisphere.

Let S be the position of the Sun on its dirunal path when its zenith distance is z, at some time in the afternoon before the sunset. The position where the Sun sets in

the angle $Z\hat{P}W = 90^o$. The ascensional difference $\Delta\alpha$ is obtained using the relation

$$\sin \Delta\alpha = \tan\phi \tan\delta.$$

$SPS_t = \theta + \Delta\alpha$ is the angle to be covered by the Sun from the given instant upto the suset. This angle divided by 15 and 6 give the time that is to elapse before sunset in hours and *nāḍikās* respectively.

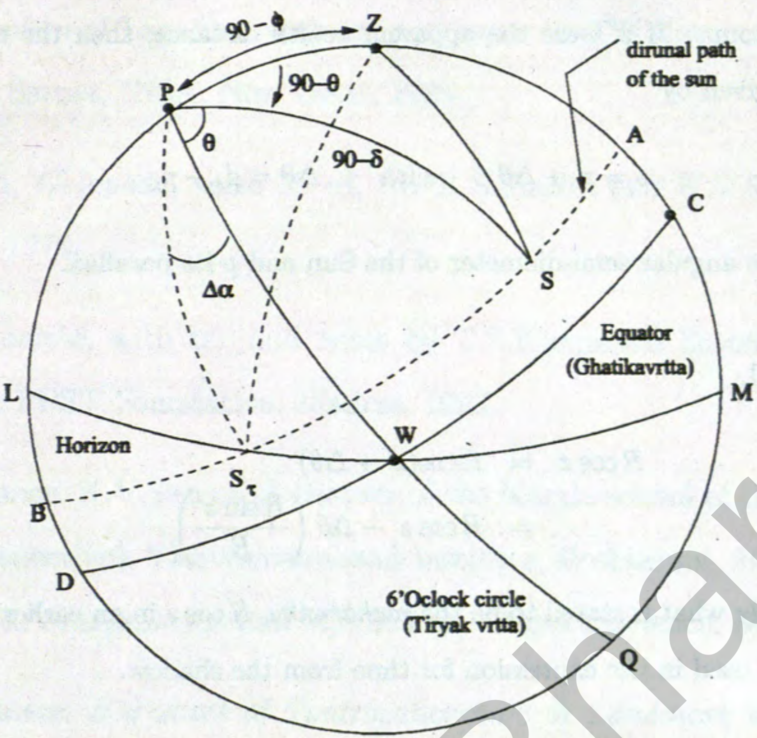

Fig.5: Determination of time from shadow measurements.

Applying the cosine formula to the spherical triangle PZS, we have

$$\cos z = \sin\phi\sin\delta + \cos\phi\cos\delta\sin\theta,$$

or,

$$\sin\theta = \frac{\cos z}{\cos\phi\cos\delta} - \tan\phi\tan\delta$$
$$= \frac{\cos z}{\cos\phi\cos\delta} - \sin\Delta\alpha.$$

Hence, the time that is yet to elapse before sunset in angluar measure is given by

$$\theta + \Delta\alpha = \sin^{-1}\left[\frac{\cos z}{\cos\phi\cos\delta} - \sin\Delta\alpha\right] + \Delta\alpha.$$

This is precisely the method expressed in the verses quoted above, for determing the time to elapse before sunset from shadow measurements. The method to determine the time elapsed after sunrise is exactly similar. It is worth mentioning here that, in finding *z*, the correction due to the finite size of the Sun and its parallax are also

taken into account. If z' were the apparent zenith distance, then the true zenith distance z is given by

$$z = z' + \Delta\theta \qquad with \qquad \Delta\theta = d_s - p,$$

where d_s is the angular semi-diameter of the Sun and p its parallax.

As $\Delta\theta$ is small,

$$\begin{aligned} R\cos z &= R\cos(z' + \Delta\theta) \\ &= R\cos z' - \Delta\theta\left(\frac{R\sin z'}{R}\right). \end{aligned}$$

This is precisely what is stated to be the *mahāśaṅku*, $R\cos z$ in an earlier verse [20], which is to be used in the expression for time from the shadow.

7 Concluding Remarks

A characteristic feature noteworthy of Indian astronomy is that, there has been a continuity in the tradition from the period of *Vedaṅga Jyotiṣa* (1400 BC) till recent times. An attempt has been made to highlight the general progress in Indian astronomy during this period, and in particular, some of the theoretical advances made in the Kerala school of astronomy. The achievements of the Kerala tradition, starting with the Madhava of Sangamagrama (b.1340 AD) are truly remarkable. As an illustration two examples were given from the work *Tantrasaṅgraha* of Nilakantha Somayaji (c.1500 AD). His another important treatise *Aryabhatīya-bhāṣya* has not yet drawn the attention of scholars. Similarly, some of the commentaries of the classic works like *Siddhantaśiromaṇi* have not been deeply studied. It is the need of the hour to look into them for innovative formulations, if any.

References:

1. *Siddhantaśiromaṇi , Adhyāya 1, Adhikāra 1, verse 9*, ed. by Muralidhara Chaturvedi, Sampurnanand Sanskrit University, Varanasi, 1981.

2. *Kṛṣṇa-yajurveda-taittirīya-saṃhitā, Kaṇḍa 1, Praśna 4.*

3. *Vedaṅga Jyotiṣa* of Lagadha, with tr. and notes by T.S.Kuppanna Sastri, cr. ed. by K.V.Sarma, INSA, New Delhi, 1985.

4. *Aryabhatiya, Golapāda*, verse 39 ed. by K.S.Shukla and K.V.Sarma, INSA, New Delhi, 1976.

5. *Pañcasiddhantikā*, with tr. and notes by T.S.Kuppanna Sastri, cr. ed. by K.V.Sarma, PPST Foundation, Madras, 1993.

6. See for instance, K.V.Sarma, A History of the Kerala school of Hindu Astronomy (in perspective), Vishveshvaranand Institute, Hoshiarpur, 1972; K.Ramasubramanian, M.D.Srinivas and M.S.Sriram, Current Science, **66** , 784, 1994;

7. See for instance, *500 years of Tantrasaṅgraha : A Landmark in the History of Astronomy*, ed. by M.S.Sriram, K.Ramasubramanian and M.D.Srinivas, Indian Institute of Advanced Study, Shimla, 2002.

8. Indian Astronomy - A source book, p 182, ed. by B.V.Subbarayappa and K.V.Sarma, Nehru Centre, Bombay, 1985.

9. See for instance, *Sūryasiddhanta, Tripraśnādhikāra*, verse 12-14, p 80-82, ed. by Krishna Candra Dvivedi, Sampurnanand Sanskrit University, Varanasi, 1987; or *Siddhantaśiromaṇi, Tripraśnādhikāra*, verse 10-17, p 158-165,, ed. by Muralidhara Caturvedi, Sampurnanand Sanskrit University, Varanasi, 1981.

10. *Tantrasaṅgraha* , *Tripraśnādhikāra*, verse 10-11, p 192, ed. by K.V.Sarma, Panjab University Indological Series-10, Hoshiarpur, 1977. For a detailed explanation of the verses refer to - K.Ramasubramanian and M.S.Sriram, Indian Journal of History of Science, **38.2**, 2003, p 129-144.

11. *Tantrasaṅgraha* , *Tripraśnādhikāra*, verse 23-25, p 200-201, ed. by K.V.Sarma, Panjab University Indological Series-10, Hoshiarpur, 1977.