



M.S-7

The Concept and Evolution of Binary Numbers

K. V. Ramakrishna Rao, B.Sc., M.A., A.M.I.E., C.Eng.(I), B.L.,
25, Venkatachala Iyer Street., West Mambalam,
Chennai-600 033

Introduction: The concept of Binary numbers must have been perceived, conceived and evolved by the people who knew the number Zero and Infinity. They must have known the rules and operations of Zero also. When and where, the people who used these two numbers lived? Scholars point out that were or are people, who use still hexagesimal system. But, they do not tell, where such people i.e, the inventors or users of Binary Numbers could have lived. With the available evidences, it is evident that they were the ancient Indians, who knew binary numbers, used them for different purposes, developed series and formulated rules for operations with zero. But, in modern context, the following questions arise specifically –

1. Whether the concept, evolution and development of Binary Numbers had been accident, incidental or a chance by the ancient Indians?
2. In what way, they were actually connected with mathematics, specifically geometry?
3. Whether they had actually formed series and Number Triangles to find out the properties of Decimal Number System and then zero in to Binary Number System as perfect to suit speech, language and communication.
4. How they conceived the two states – 0 and 1, false and true, non-existence and existence and so on?

To get answers and assess the reality, the Vedic literature has to be analyzed.

Two States of Existence: A Rigvedic hymn significantly elaborates two states of existence by illustrating with different examples. The entire discourse is covered under the expression "Na Sat Na Asat" (in the Hymn of Creation, Rigveda Samhita, Mandala.10 Verse.129). A translation of the hymn goes like this (<http://www.geocities.com/augustfour/vedic.html>). "At first was neither Being nor Nonbeing. There was not air nor yet sky beyond. What was wrapping? Where? In whose protection? Was Water there, unfathomable deep? There was no death then, nor yet deathlessness; of night or day there was not any sign. The One breathed without breath by its own impulse. Other than that was nothing at all. Darkness was there, all wrapped around by darkness, and all was Water indiscriminate, Then that which was hidden by Void, that One, emerging, stirring, through power of Ardor, came to be. In the beginning desire arose, which was primal germ cell of mind. The Seers, searching in their hearts with wisdom, discovered the connection of Being in Nonbeing. A crosswise line cut Being from Nonbeing. What was described above it, what below? Bearers of seed there were and mighty forces, thrust from below and forward move above. Who really knows? Who can presume to tell it? Whence was it born? Whence issued this creation? Even the Gods came after its emergence. Then who can tell from whence it came to be?"



That out of which creation has arisen, whether it held it firm or it did not, He who surveys it in the highest heaven, He surely knows - or maybe He does not!"

The Satapata Brahmana (XI.I.6.1 to 4) explains the creation of one syllable and two syllables from the beginning.

1. In the beginning, there was water. The water getting heated (by toiling and fervid action) and Hiranya Garba / Golden Egg was produced in a year. (It is comparable to a sperm successfully piercing a Ovum and fertilization taking place. Zygote forms, fetus develops. After the gestation period, the baby is born).
2. Indeed, in the next verse, the birth of a man (Prajapati) is mentioned.
3. At the end of the year, the child tried to speak uttering "bhuh", "bhuvah" and "svah" and they became earth, air and sky respectively.
4. Thus, when a child starts speaking, he speaks one syllable and two syllables.

Here, the syllables "bhuh", "bhuvah" and "svah" are the numbers 1, 0 and ¥. The earth (1) is existing, so also air (0) and sky (¥). We can feel air, but not sky, and approach sky. Thus, 0 and 1 are two levels of existence and the third state is incomprehensible, and that can only be approached and not attained. Thus, any value can approach ¥ but cannot attain ¥. Thus, the Vedic literature repeatedly mentions about two states and they are -

1. The State of existence and
2. The State of non-existence.

In fact, it is said that there is neither the state of existence nor the state of non-existence, which is nothing but acceptance of only two states.

Thus, under these two categories all possibilities are covered.

1. **The State of Existence:** on, positive, yes, is, day, good, true, visible, life, death, normal and so on.
2. **The State of Non-existence:** off, negative, is not, night, bad, false, invisible, death, deathlessness, abnormal and so on.

The pairs explicitly and implicitly mentioned are -

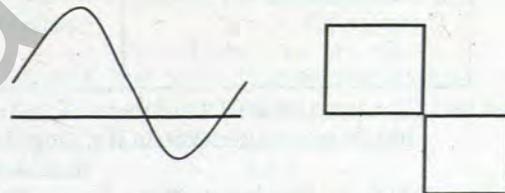
1. Being and Nonbeing (Existence and Non-existence).
2. Air and Sky beyond (Boundaries of Universe).
3. Above and below (Sky limits).
4. High and low (Water levels).
5. Death and deathlessness (limits of life).
6. Night or day (counting days).
7. Breadth or breathlessness (Counting life).
8. Void and Emergence/complete (Yes or No).
9. Abandon/hate and Ardor/desire (Like or Dislike).
10. Knowing or unknowing (True or False).

Why Vedas should convey positive meaning in negation? *Neti, neti* (Not this, not this) is related to *Na Sat Na Asat*. In fact, this is realization of the Absolute (here OM). It is achieved by way of Negation and Affirmation. *Neti, neti* (Not this, not

this) of the Hindu teachings is the way of Negation. The Absolute is Absolute, because, others are relative only. The Relative can approach Absolute, but it cannot become Absolute. In short, it negates everything that is not Absolute. Hence the Absolute is the last remainder. Therefore, for everything, only reality is held considering the two possibilities.

1. The above list also points out the significance of AND and OR used in the pairs.
2. First, AND is emphasized and then, OR depending upon the states, levels, limits or comprehension of such conditions.
3. Whether, such usage has been accidental or incidental is debatable, but the usage is factual. As such usage has also been in connection with the explanation of Two State of Existence, the significance is appreciable.
4. Again, as it is used in negation - NOT, their negative pairs are also implied i.e, combination of NOT with AND and OR leading to NAND and NOR, and of course, XOR and XNOR.
5. And these are used in logical circuits to perform a particular logical function based on the concepts of 'and', 'either-or', 'neither-nor' etc.
6. Therefore, in mathematics and mathematics-related problems, only two possibilities are there in spite of logic, philosophy and other processes involved. For one problem, two answers cannot be obtained.

In Indian context, even the negative aspect is viewed positively. Thus, death is considered with deathlessness. If we take a curve, such states are easily understood.



The sinusoidal curve represents the condition of a current flow in a cycle. During the negative cycle, even though, it flows, as it is on the negative side, the curve does not appear on the positive side, but current continuous to flow. Similarly, the conditions of binary numbers, they continue to exist, even though, they are 0 and 1 and zero is there. Here 0 never indicates any non-existence state. That is why the Vedic logistics emphasized about the reality of "non-existence" to appreciate its existence in linguistic and mathematical processes. Without understanding the concept, evolution and existence of 0, the formation of number system cannot be understood. Therefore, if any 'number system' had been there without 0, it is evident that such 'number system' with represented 'numbers' were used just like anybody for counting without any logic, philosophy or bio-rhythm. To make one understand the incomprehensible, abstract or gross in comprehensible, concrete or gross, symbolism is required. The symbolism is also not just accident to represent things like Egyptian hieroglyphs, Babylonian cuneiform or any other pictograph / pictoglyph. Thus, representation of conceived 'zero' in number symbol "." or "0" has been the greatest invention ever made. As some scholars interpret, the usage or representation of zero as "." and / or "0" is not any defi-



ciency, but the implied arithmetical and geometrical definition for the geometrical point and mathematical zero. In a space / plan, if something has to be objectively symbolized, the starting point is a "point" and so also in number system. This only leads to the conception of negative numbers, co-ordinates, two dimentions, three dimensions, orthography

and so on.

The Basis of the Choice of 0 and 1: Everybody knows that the binary numbers are 0 and 1, but, how, why, for what they have been chosen etc., are not explained specifically and properly. Let us consider the following proportions:

$0^0 = 0$	$0^0 = 0$ or 1, as 0 is treated as a number. If any number power is equal to 1, then, $0^0 = 1$, but, the value of $0 = 0$ only, i.e, the value of the number $0 = 0$ only. Therefore, $0^0 = 0$ only and not 1.
$0^1 = 0$	
$1^0 = 1$	
$1^1 = 1$	Similarly, $0^1 = 0$ is explained.
$2^0 = 1$	
$2^1 = 2$	In theory of indices, it is proved that $n^0 = 1$. Therefore, $1^0 = 1$ and $1^1 = 1$ are understood.
$3^0 = 1$	
$3^1 = 3$	The other number powers and the results are covered by the general notation $n^0 = 1$ and $n^1 = n$.
.....	
.....	Thus, the only two numbers, which have the same states of equalities, are 0 and 1.
$n^0 = 1$	
$n^1 = n$	Is it coincidence or providence or predetermination? Here only, lies the secret.

$n^0 = 1$ is not explained properly: Take any Algebra book explaining exponentials and theory of indices, $n^0 = 1$ is explained away. Typically, it is mentioned, shown or proven as follows:

<p>We know, $a^m \times a^n = a^{m+n}$, Putting $m = 0$ and $n = n$, It becomes, $a^0 \times a^n = a^{0+n} = a^n$ Therefore, $a^0 \times a^n = a^n$ Or $a^0 = a^n / a^n = 1$</p>	<p>Here, in the expression, $a^{0+n} = a^n$, a^0 is ignored, evidently assuming that it is equal to 1. In fact, that only has to be proved. If a^0 exists, then, $a^{0+n} = a^0 \times a^n$ only. Thus, unwittingly, mathematicians equate equals to get the desired answer / proof.</p>
--	--

Or it is asserted that "The zero power of a number $a \neq 0$, is by definition equal to unity". Some mathematicians claim that 0^0 has no sense, however, in the context of proving $n^0 = 1$, they proceed like this,

If $a \neq 0$, then, $a^0 = 1$.

If $a = 0$, then, $0^0 = 0$, which they say has no sense. But, in binary context, it has unique meaning and that has been explained above.

<p>5 4 3 2 1 -5 -4 -3 -2 -1 0 1 2 3 4 5 -1 -2 -3 -4 -5</p>	<p>Therefore, 0 is a number. It is neither +ve or -ve in its state, but it is either +ve or -ve (existing in two states), particularly at the origin or at the centre point in the co-ordinates. It is +ve, when it is dealt with +numbers and -ve, when with -ve. Therefore, there is -0 and +0. Such co-ordinates may be represented as <u>-0 0 +0</u> with the implication $0 < +0, 1$ and $-1 < -0 < 0$. When this is extended to ordinary numbers, we have co-ordinates (x,y) with numbers around zero as origin.</p>
--	---



By understanding the existence of number 0 and its association with 1 and the unique property applicable only to them, we are tempted to define "binary numbers".

Binary Numbers: Binary Numbers are the two numbers, which are obtained, when they are raised to their respective powers of the same value to get the same numbers as equivalent (definition by the author).

If we try with 0 to 9 numbers, as tried above, only 0 and 1 satisfy the conditions. Therefore, 0 and 1 are the only two numbers, which are obtained, when they are raised to their respective powers of the same value, i.e. $0^0 = 0, 0^1 = 0, 1^0 = 1$ and $1^1 = 1$. This is further confirmed from the Binomial Triangle. $(a + b)^0 = 1$ explains the two states, when a and b are taken as two variables. Then, $(a + b)^1 = a + b$ with the co-efficients 1 and 1. Though, we start with 0, the series starts with 1. Such terms, i.e, the given variables raised to 0 comes in the beginning and the end, thus, we get 1 and 1 in the beginning and end.

Binomial expressions for powers of 0, 1, 2, 3, 4, 5, 6,	The Triangle formed with Co-efficients
$(a + b)^0 = 1$	1
$(a + b)^1 = a + b$	1 1
$(a + b)^2 = a^2 + 2ab + b^2$	1 2 1
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	1 3 3 1
$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$	1 4 6 4 1
$(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$	1 10 5 5 10 5 1
$(a + b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$	1 6 15 20 15 6 1
.....	

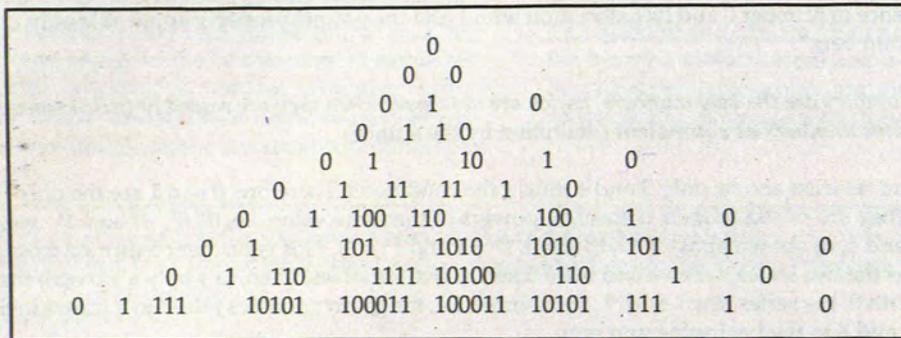
Here, it has to be noted that 0s are conveniently ignored. If they are added, the Triangle would appear as shown below. From this, naturally, we can think of a Binary Triangle, that is nothing but Pingala Triangle, defined by him in his treatise Pingala Sastra.

Binomial Triangle with co-efficients ignoring 0s done conventionally	Binomial Triangle with co-efficients with 0s shown to bring out the significance
1	0
1 1	0 1 0
1 2 1	0 1 1 0
1 3 3 1	0 1 2 1 0
1 4 6 4 1	0 1 3 3 1 0
1 5 10 10 5 1	0 1 4 6 4 1 0
1 6 15 20 15 6 1	0 1 5 10 10 5 1 0
	0 1 6 15 20 15 6 1 0

If a triangle is constructed only with binary numbers, first with AND addition and then with OR addition, we get Triangles as follows:

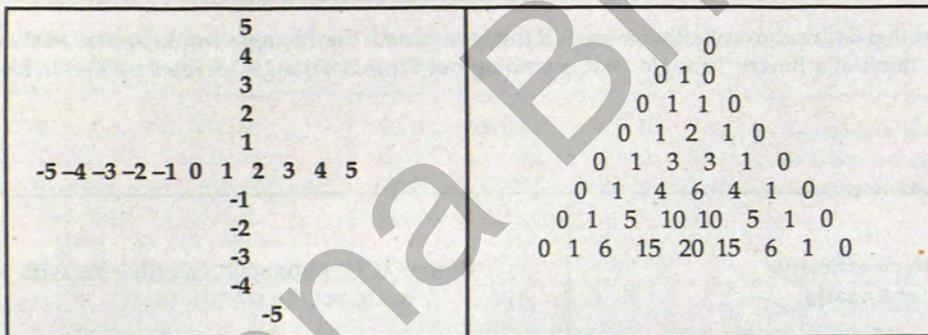
AND Triangle	OR Triangle
0	0
0 0	0 0
0 1 0	0 1 0
0 0 0 0	1 1 1 1
	1 1 1 1 1 1
	1 1 1 1 1 1

Thus, this is continued, because of OR addition. Thus, a Triangle would be constructed only with Binary numbers using Binary addition as follows:



This is similar to that one shown above, if Binary numbers are replaced with Decimal numbers.

Binary Co-ordinates: Incidentally, the concept of Zero, Numbers 1, 2, 3, etc., and √ leads to the formation of co-ordinates in two and three dimensions for construction purposes. As it has to be noted that all these exercises carried out by the Vedic scientists had been for the recitation of Vedic hymns with prosody, manufacture of bricks trephzoids of different sizes and shapes, construction of altars, conduct of experiments with chemicals in yagna and so on. But, the experimenting with numbers and construction signifies the origin and end of the construction. Thus, the co-ordinates with zero had been marked by them. The arrangement of -ve and +ve number with zero as centre on the floor or a plan leads to co-ordinates in two dimensions, which have now been designated as x and y axes. The arrangement of numbers with zero as centre in space later filled with bricks and others leads to the formation of three-dimensional co-ordinates, which have now been designated as x, y and z axes. But, all these have been incorporated in the Meruprasthana in Number Triangle and as well as building (Meru) constructed. The numbers written with zero and the Pingala Triangle with Zeros are shown for appreciation below:



Another important significance is that this representation is nothing but the top view of the Pingala / Binomial Triangle from the top. This is again similar to simple Sri Chakra (drawn in two dimensions) and Mahameru (viewed in three dimensions). Now, we draw such Ds and co-ordinates only with 0 and 1 and they are depicted as follows:

AND D	OR D	Binary Co-ordinates	
0	0	+1	The AND and OR triangles signify not only the Binary Number System, but also its consistency with the method followed. Thus, the binary co-ordinates visualized and depicted have been consistent with the Binary Number System and also with Pingala Sutras.
0 0	0 0	+0	
0 1 0	0 1 0	-1 -0 0 +0 +1	
0 0 0 0	1 1 1 1	-0	
		-1	

The Uniqueness of Number 2: The Binomial expansion, series and Triangle expounded the importance and unique quality of number 2. The sum of binomial co-efficient in each series is equal to 2^n . Pingala has noted this uniqueness and thus, he conceived the two properties of number 2. It starts with 0 in power and thus not figuring in the Triangle, but, 1 appears continuously in the beginning and end in the series. Thus, the triangles appears to be covered or crowned with 1 and all other co-efficients are arrested in its trinagular-matrix, that is beautifully and mathematically named as Meruprasthana. The number 2 nails down the numbers 1 and 0.

In fact, why only Binomial theorem, binomial series, binomial triangle etc.? Why not Trinomial? It is not known as to anybody tried with such proportion. Of course, there is a mention about 'Trinomial Triangle', but it is not with three variables (www.mathworld.wolfram.com).



$(a+b+c)^0 = 1$
 $(a+b+c)^1 = a + b + c.$
 $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca.$
 $(a+b+c)^3 = a^3 + b^3 + c^3 + 3 a^2b + 2 ab^2 + 2 b^2c + 2 b c^2 + 6 abc.$

1	2^0	1
1 1	2^1	2
1 2 1	2^2	4
1 3 3 1	2^3	8
1 4 6 4 1	2^4	16
1 5 10 10 5 1	2^5	32
1 6 15 20 15 6 1	2^6	64
1 7 21 35 35 21 7 1	2^7	128
1 8 28 56 70 56 28 8 1	2^8	256
1 9 36 84 126 126 84 36 9 1	2^9	512
1 10 45 120 210 252 210 120 45 10 1	2^{10}	1024
1 11 55 165 330 462 462 330 165 55 11 1	2^{11}	2048
1 12 66 220 495 792 924 792 495 220 66 12 1	2^{12}	4096

Therefore, it is clear that the series do not form any binary pattern / matrix useful to binary computation. Thus, the relation of Binomial numbers to Binary numbers have been well established and this is found only in the Pingala's Chand Sastra.

Leonardo Fibonacci's Manipulation: Leonardo Fibonacci obtained Indian mathematical, astronomical and medical manuscripts and charts through Arabic merchants. The instantaneous knowledge obtained on the platter inspired him to exhibit as the inventor of the series. However, in the so called "Fibonacci series", he committed a blunder and exposed himself. As he could not understand the significance of zero, he left the zero in the beginning of the series.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597.....

But, here, the important questions are not answered:

1. Why the first number is 1?
2. Why there are two 1s?
3. What is the first number with which 1 is added to get 1?

The first number must have been 0, so that when it is added with next number 1, we get $0 + 1 = 1$. Thus, the series is explained. But, the poor Fibonacci had been a fibber to swallow zero!

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597.....

That the 1 is repeated must have exited any mathematician, to fathom into its significance to find out the truth. Consider the series,

0, 1, 1, 2.....

Thus, the 2 covers 0, 1, 1, as in the Pingala Triangle.

Suppose, if were asked to write a "Fibonacci series" only using 0 and 1 (pardon Fibonacci for he did not know what he was doing), we would have written 0, 1, 1 and stopped. Of course, it would have continued like this, 0, 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111 and so on. Thus, it is not a mere repetition of 1, but the Binary concept ingrained in it. Anyway, Leonardo Fibonacci and Nemorarius have been credited for introducing the 'al-sifer' or "Sunya" in Europe during Italian renaissance!

To answer the above questions:

Why the first number is 1? The first number is not 1 but 1.

Why there are two 1s? There are two 1s, as the first number is 0 and it signifies the Binary Number System is inherent in these two numbers 0 and 1. Had Fibonacci anticipated or proceeded further, George Boole need not have learnt classical languages to understand logic and apply it to Mathematics to re-invent his algebra.

What is the first number with which 1 is added to get 1? ZERO! The Origin of all Numbers.

Why Use only Not-Gates – NOR, NAND, NOR? Why do not we use YES OR, YES AND and YES OR gates, but use



only NOR, NAND and NOR? In Binary computation, we always hear about logical gates – Not-OR = NOR, Not-AND = NAND and Not-OR = NOR. Because, logically, we know, that non-existence is a state of existence. This is exactly ingrained in “na sat na asat” and “neti neti” concepts as explained above.

Pingala Showed that All Numbers were Covered under Binary System:

Pingala, through his simple aphorisms proved that all other decimal numbers other than 0 and 1 are covered under Binary System, as they can be reduced to in terms of 0 and 1. Though, he was dealing with metrics and prosody, he dealt with syllables, which were based on sound and vibrations. Pingala did not have a computer like the one we have, however, he had the same binary computing methodology, what we use or our computer uses. Like him, all were using for prosody and Vedas and chanting hymns sinusoidally and uttering syllables in binary.

Western Scientists Treated Binary Numbers theoretically, before Put into Practice:

Just, review the controversy of ‘dual nature of matter’, ‘matter waves’, etc. How, the equation, $E = mc^2$ is explained without mincing words? The mass-energy equivalence actually accepts the conversion of mass into energy and vice versa. The visible is made to Invisible and vice versa! Has any scientist shown such conversion visibly? Has any electrical scientist shown the Electricity? Einstein has also declared that philosophy starts where science ends! The very word “ATOM” proved wrong, and it got split to produce many subatomic particles. Dalton believed that that it could not be split and hence christened “atom” = $a + tom$ = that cannot be divided or split into. Though, he was proved wrong, the name became misnomer, it continued. The ‘Theory of Relativity’ argues that everything is relative and there is not absolute. But, there is neither relative nor absolute. Psychologists assert that none is completely normal or abnormal, i.e, none is neither normal nor abnormal.

Conclusion: From the above discussion, the following conclusions are drawn:

1. Concept, evolution and formation of ‘number system’ with ‘0’ have been the greatest ever invention.
2. The thorough study of syllables, words and their association with numbers lead to formation of number series, later represented as Number Triangles.
3. The “Binomial series”, “binomial theorem”, binomial triangle” etc., with two variables played a crucial role in the Vedic period, thus, Pingala immediately codified it in Chandas.
4. The concept, evolution and application of the two states existence and non-existence have been consistent with Vedic literature and continuous.
5. The Rigvedic description of Creation spells out step by step development of logic of existence and non-existence with only two possibilities.
6. The unique properties of 0 and 1 made them to be qualified for “binary numbers” and a definition for “binary numbers” is proposed perhaps, for the first time.
7. The concept and evolution of “binary number system” also led to the conception of negative numbers, C0-ordinates with origin (0,0) etc.
8. The formation of Pingala / Binomial Triangle with zeros led to the thinking of Binary Triangles using 0 and 1.
9. The AND Triangle (using AND addition) and OR Triangle (using OR addition) give unique results to understand the logic behind gates. The “Binary Binomial Triangle” constructed reveals the concept of “Meru” and also leads to “B