



M.S-10

MAGIC SQUARE

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SYNOPSIS

India has a rich culture heritage, which is preserved in her Sanskrit literature. Her hoary wisdom is contained in it. This is maiden attempt to make use of Vedic formulae to arrive the sum /product of Magic Squares & hope everyone will appreciate this paper.

History and meaning

Magic Squares have identical number of rows & columns.

You might have come across such squares filled by numbers in its 16, 25... houses so that the sum of the numbers along of the horizontal rows, vertical columns or diagonals will be same.

It is an old art and various ways in eastern & western countries are known since long.

In India, Bhaskaracharya (12th century AD) Narayan Pandit (14th Century AD) has written verses to frame such squares. There has been a square of 16 houses inscribed in Khajurao caves.

In this session, we will learn

How to fill up these squares of sum and product.

The sutras and their meaning used to construct the Magic Squares.

VEDIC FORMULAE

The resurgence of interest in Vedic Mathematics came about as a result if Jagadguru Shankaracharya of Puri Math Swami Bharati Krishna Tirthji Maharaj (1888-1960) publishing a book on the subject in 1965 which is an alternative system to the present monotonous system for solving mathematics problems. The system is based upon 16 Vedic Formulae in Sanskrit from the appendix of Atharvaveda.

Swamiji has said, "Any mathematics branch – pure or applied – comes in the jurisdiction of these formulae".

Ekadhikena Purvena

This is the first Vedic sutra used for the construction of Magic Square of sum meaning "By one more than the previous one". e.g. one more than 14 is 15, One more than 68 is 69 and so on.

In this context, it can be an arithmetic series with common difference 1.

Starting with any term a & common difference d , an arithmetic series (AP) can be written with the required terms $a, a+d, a+2d, a+3d...$

e.g. if $a=3, d=2, AP = 5, 7, 9, 11...$

These terms are filled up in the houses of a square by a special Vedic method based on *Paravartya Yojayeta Sutra*

Paravartya Yojayeta

This sutra is used for filling up the numbers in the houses of Magic Square

In this context, when the number is outside the square, it is said to be 'transposed' to be taken inside the square and 'adjusted' means that number is written at the opposite end in a row or column.

Anurupyen

This sutra is used for obtaining numbers when Magic Square of product is to be built up.

Proportionately

The numbers are selected in proportion using combination of ratios.

i.e. the numbers in geometric progression

($a, ar, ar^2, ar^3...$)

where

a is the first term

r is common ratio

The numbers are chosen by maintaining horizontal common ratio (CRH) and vertical common ratio (CRV) starting with 'a' common first term

TYPES OF MAGIC SQUARES

1. MAGIC SQUARE OF ODD HOUSES
2. MAGIC SQUARE OF EVEN HOUSES

The numbers of rows & columns in a magic square are always identical.

MAGIC SQUARE OF ODD HOUSES

This is a 3×3 square or of 9 houses

The procedure for filling the nine houses is common for both the sum and product.

Magic Square of Sum

Select an AP with 1 as first term & 1 as common difference AP of 9 terms will be natural numbers 1, 2, 3... 9.

Step 1. Draw a square of 9 houses & imagine 3 diagonal lines

Step 2. Start from lower diagonal & first term of AP series, write 3 consecutive

Numbers of AP series on each lower, middle & top diagonal in order

Upwards to the right

Step 3. Out of these 9 numbers, 4 numbers appear outside the square.



Transpose these numbers inside the square

- 1 from bottom to top house
- 3 from right to left house
- 7 from left to right house
- 9 from top to bottom house

Step 4. Now, after transposing the outer terms inside the square the square look

Like

The magic square of sum 15 is ready.

The sum of each of the horizontal rows, vertical columns and diagonals is 15

Any number of such magic square of different sums can be constructed by choosing appropriate AP of 9 terms. The common sum in the square of nine houses will be 3 times the number appearing at the center house of that square. If a is the first term and d is the common difference of AP the sum along vertical, horizontal & diagonal house will be $3(a+4d)$

Construct a 9-house magic square

Group 1) $a = 20, d = 1$ Group 2) $a = 5,$
 $d = 3$ Group 3) $a = 7, d = 2$

MAGIC SQUARE OF PRODUCT

Prepare a chart of 3×3 terms by maintaining required CRH & CRV

Write the 9 terms if horizontal ratio CRH = 1:3 & CRV = 1:2

CRH 3			order
1	3	9	(1 to 3)
2	6	18	(4 to 6)
4	12	36	(7 to 9)

Step 1. Draw a square of 9 houses

Step 2. Start from lower diagonal & first term of GP series, write 3 consecutive

numbers of GP series on each lower, middle & top diagonal in order

upwards to the right

Step 3. Out of these 9 numbers, 4 numbers appear outside the square.

Transpose these numbers inside the square

- 1 from bottom to top house
- 9 from right to left house
- 4 from left to right house
- 36 from top to bottom house

Step 4. Now, after transposing the outer terms inside the square the square look

Like

The magic square of product 216 is ready.

Product of numbers in horizontal rows $X \times X \times X$
 Vertical columns
 Diagonals

will be 216

This product will be the cube of number appearing @ the center i.e. 6^3

If a is the common first term horizontal & vertical GPs having r & k as respective common ratios, then for 3×3 square, the VHD product will be $a^3 (rk)^3$

In the above example

$a=1, r=3, k=2$

VHD product is $1^3(3 \times 2)^3 = 216$

Construct a 9-house magic square of product

Group 1) $a = 1, CRH = 4, CRV = 3$ VHD
 $= 1^3(4 \times 3)^3 = 1728$

Group 2) $a = 2, CRH = 3, CRV = 2$ VHD
 $= 1^3(3 \times 2)^3 = 1728$

MAGIC SQUARE OF EVEN HOUSES MAGIC SQUARE OF 16 HOUSES

This is the smallest magic square of even number of houses. It is also known as 4×4 square or a square of fourth order.

We will discuss construction of magic square of 16 houses. The Vedic sutra "transpose & adjust" is used but the meaning of transpose is taken as clockwise to anticlockwise directions and vice versa.

There are two methods for magic squares of sum. But we are going to deal with only one method used for both sum and product.

Before proceeding for construction of magic square of 16 houses, let us know the meaning of transpose clockwise to anticlockwise.

Draw 4×4 square of 16 houses and divide equally in 4 sub-squares such that each sub-square will contain one corner of square ABCD. Accordingly let us name these sub-squares as A, B, C & D.

Now each sub-square contain 4 houses which are named as per their position as LT - Left Top; RT - Right Top; LB - Left Bottom & RB - Right Bottom houses.

Let us consider an AP series 1, 2, 3, ..., 16 see how the four groups of this series Group 1 (1-4); Group 2 (5-8); Group 3 (9-12) & group 4 (13-16) are filled up.

To understand the procedure for filling these 16 houses, let us consider 16 dots for 16 houses and accordingly fill 16 houses.

Diagram of Dots

First and fourth groups start from 1st houses first and fourth row respectively in ACW and CW directions in these rows only. Group 2 and 3 start from last house of 3rd and 4th row respectively in ACQ and CW directions in these rows only.



Let us fill up the first two groups in ACW direction.

Diagram *

Let us fill up the last two groups in CW direction.

Diagram *

Now, the combination of all 4 groups and thus the magic square!

The VHD sum is 34, along with the same sum in four sub-squares;

four houses at the center;

four corners of the square;

2nd and 3rd house of 1st row & 4th row;

2nd and 3rd row 1st and 4th column;

1st row 2nd column - 2nd row 4th column - 3rd row 1st column - 4th row 3rd column;

1st row 3rd column - 2nd row 1st column - 3rd row 4th column - 4th row 2nd column;

In general, for 4 x 4 square, the VHD sum will be obtained by $(4a + 30d)$ where a and d are respectively first term and common difference of AP.

Activity

Construct 4 x 4 and fill AP 8,9,10 to 23 of 16 terms, the VHD sum will be $[(4 \times 8) + 30] = 62$

Construct 4 x 4 and fill AP 1,3,5 to 31 with a=1, d=2 of 16 terms, the VHD sum will be $[4 + (30 \times 2)] = 64$

MAGIC SQUARE OF EVEN HOUSES OF PRODUCT

Prepare a chart of 4 x 4 terms by maintaining required CRH & CRV

Write the 4 terms if horizontal ratio CRH = 1:3 & CRV = 1:2

	CRH 3			Group
1	3	9	27	1
2	6	18	36	2
4	12	36	108	3
8	24	72	216	4

Step 1. Draw a square of 16 houses

Step 2. 1st and 4th groups start from 1st houses first and fourth row

respectively in ACW and CW directions in these rows only.

Group 2 and 3 start from last house of 3rd and 4th row respectively in

ACW and CW directions in these rows only.

Let us fill up the first two groups in ACW direction.

Diagram *

Let us fill up the last two groups in CW direction.

Diagram *

Now, the combination of all 4 groups and thus the magic square of product!

The VHD product is $(3 \times 2)^6 = 46656$,

along with the same product

in four sub-squares;

four houses at the center;

four corners of the square;

2nd and 3rd house of 1st row & 4th row;

2nd and 3rd row 1st and 4th column;

1st row 2nd column - 2nd row 4th column - 3rd row 1st column - 4th row 3rd column;

1st row 3rd column - 2nd row 1st column - 3rd row 4th column - 4th row 2nd column;

If a is the common first term horizontal & vertical GPs having r & k as respective common ratios, then for 4 x 4 square, the VHD product will be $a^4 (rk)^6$

In the above example

a=1, r=3, k=2

VHD product is $1^4 (3 \times 2)^6 = 46656$

MIRROR MAGIC SQUARE OF 16 HOUSES

The 16 houses are filled in such a way, that not only Vertically, Horizontally, Diagonally sum but the sum in each subsquare of four houses at the center and four houses of the corners of the square will be same! As well as the same numbers may be replaced among themselves then also the sum will be same!! Amazing Magic Square!!!

98	37	25	82	= 242 =
89	73	52	28	
43	64	76	59	= 242 =
34	46	67	95	
62	45	57	78	= 242 =
26	54	75	87	
39	96	84	23	= 242 =
93	69	48	32	
=	242	242	242	242
242	242	242	242	

Reference: "Vedic Mathematics" Sri Bharathi Krishna Thiraji
MAGIC SQUARE by Vedic Methods - V G Unkalkar